DIVERGING MATHEMATICS

Sample Adventures in Clock Algebra

The year starts with the expedition described on this page. Additional adventures, like the ones outlined on the second page, will be organized as time permits and interest dictates, or perhaps become the foundation of future seminars on this topic.

The seismic shift in this seminar is to wrap the number line around in a circle, and the household item that you are familiar with that does this is a clock.

To ease into how this twists things around, focus on just the hour hand of a 12-hour clock and only on whole hours (that is, we really just care about the integers getting wrapped around the circle). Consider the following questions:

1. (Addition) Does it make sense to add?
   (a) What is 4 + 9? (Think: What time is 4 hours after 9:00?)
   (b) What is 7 + 8?
   (c) What is 1+2+3+4+5+6?

2. (Subtraction) Does it make sense to subtract?
   (a) What is 4 - 9? (Think: What time is 9 hours before 4:00?)
   (b) What is 7 - 8?
   (c) Did we really need this? Instead of subtracting 9, what could you add?
   (d) Solve: 10 + x = 3

3. (Multiplication) What does multiplication mean now? Thinking back to elementary school, you probably learned "skip-counting", for example counting by 3s.
   (a) Count by 3s starting at 5:00.
   (b) Count by 5s starting at 3:00.
   (c) Count by 9s starting at 5:00.
   (d) Complete a "times table" for multiplying on this clock.

We will start our first meeting by discussing these questions and sharing discoveries and questions that you encountered along the way.

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Additional Adventures:

Here is an incomplete list of ideas for additional expeditions that wrap numbers around a circle.

1. How do the patterns on the multiplication table for the 12-hr clock relate to the properties of the number 12? We will need to test any hypotheses by looking at clocks with a different number of hours, like maybe a 15-hr clock, or a 7-hr clock. Also, there's a nicer way to realize the 15-hr clock: wrap it around a torus.

2. Division created problems before, requiring us to learn about fractions. What problems are created here? Do fractions still fix the problem?

3. Solving linear equations is a whole different ball game. Not only are we only interested in integer solutions, but we are also wrapping the number line. The challenge is worthwhile as there are many powerful and enlightening results that will change how you think about numbers. Understanding quadratic equations is also possible, but not for the faint of heart.

4. Did you ever learn divisibility rules? You will now be able to understand them, as well as a more powerful way to use them.

5. Suppose we want to look at *all* numbers, not just integers? This is the setting for one of the simplest models of **chaos**. We will need to understand what this means *mathematically* as we try to untangle the consequences.