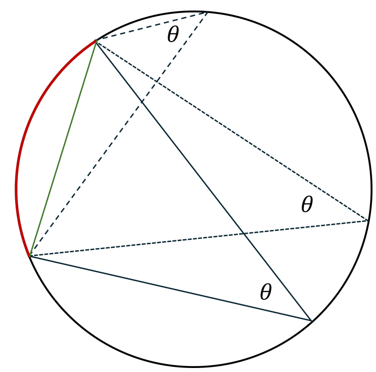
DIVERGING MATHEMATICS

Sample Adventures in Advanced Trigonometry: Ptolemy to Fourier

The year starts with the expedition described on this page. Additional adventures, like the ones outlined on the second page, will be organized as time permits and interest dictates, or perhaps become the foundation of future seminars on this topic.

Our story starts with Ptolemy, an ancient figure who is best known for his geocentric model of the cosmos - hopelessly clinging to even more ancient dogma has made him a frequent punching bag in the sciences. We will see a different side of the story - of someone who was millennia ahead of his time.



Let's start with a theorem you saw in Geometry, probably without much fanfare:

<u>Inscribed Angles Theorem:</u>

> The measure of an angle inscribed in a circle is the length of the intercepted arc divided by the diameter.

Modern statements often include the central angle and use the radius, but the most impressive part of this theorem is that the orientation of the angle in the circle is immaterial: it will always intercept the same amount of the circumference.

Inscribe a right triangle with sides a and b and hypotenuse c into a circle and consider the following questions.

1. What does the Inscribed Angle Theorem tell you about the diameter of this circle?

2. What does the Inscribed Angle Theorem tell you about the length of the arcs in this circle? Suppose the angle θ intercepts an arc of length s. What's the diameter?

3. An arc also defines a chord. Suppose the angle opposite a has measure α. What's the diameter?

4. The triangle with sides 7, 15, and 20 can be inscribed in a circle with diameter 25. Use the relationship from the previous question to find the measure of the angle opposite the side of length 7.

Extra things to ponder:

1. Does this look like one of the trigonometric "Laws"?

2. Suppose the diameter has unit length. How are opposing sides and angles related now?

3. What is the smallest circle with integer diameter that can circumscribe an *acute* triangle with integer sides?

We will start our first meeting by discussing these questions and sharing discoveries and questions that you encountered along the way.

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Additional Adventures:

In addition to the curious observation from the adventure started on the previous page, there is already the philosophical framework that everything happens within a circle. Here is an incomplete list of ideas for additional expeditions inspired by the work of Ptolemy and Fourier:

1. How do other trigonometric functions appear in Ptolemy's circle? Do any identities become easier to understand in this framework? So many new insights await the eager explorer!

2. Ptolemy's most ambitious project was to predict the motion of the cosmos, but circular orbits were inadequate. Borrowing from other astronomer, he took the idea of circles orbiting circles to new heights. Even with just these two circles, the accuracy of his model remained the standard for centuries *after* Newton's Gravity correctly characterized the orbits as ellipses (elliptical arcs are too hard to calculate!).

3. Circular orbits are easily modeled by complex numbers. We will infuse Ptolemy's work with this modern invention and re-imagine his work within this framework.

4. What is a vector? What can you do with it? We will look carefully at this and discover that the abstract concept of vectors will give us another lens to see Ptolemy's work. This view will look more like Fourier's discovery and launch us into a new direction.

5. Modern Spectroscopy and Imaging depends heavily on techniques rooted in this material. It's how Helium was discovered on the Sun before it was found on earth. How organic chemists know what molecules they have synthesized in their beakers. It's how MRI images are formed and how the Federal Reserve produces "seasonally adjusted" economic indicators.